

TOTAL TEMPERATURE THERMOCOUPLE PROBE BASED ON RECOVERY TEMPERATURE OF CIRCULAR CYLINDER*

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Abstract—A total temperature probe based on the recovery temperature of a circular cylinder is described which combines the advantages of both hot wire and shielded thermocouple probe. It is mechanically very simple and durable and may be used in low and high speed flow, from continuum to the free molecule regime and is suitable for wake and boundary layer investigations with large total temperature gradients. As an example of the use of the probe, measurements of total temperature in the hypersonic near wake of a cold cylinder (77°K) are discussed.

NOMENCLATURE

d , cylinder probe diameter;	η , recovery factor, T_{aw}/T_0 ;
D , cylinder model diameter;	μ , viscosity;
h , convective heat transfer coefficient, $q/(T_w - T_{aw})$;	ρ , density.
k , heat conductivity;	Subscripts
l , length of probe cylinder;	c , continuum flow;
M , Mach number;	f , free molecule flow;
Nu_0 , Nusselt number, hd/k_0 ;	H , hypersonic flow;
p , static pressure;	o , stagnation condition of air flow;
p_p , Pitot pressure;	s , support;
Pr_0 , Prandtl number, $c_p\mu_0/k_0$;	w , wall condition;
q , heat transfer rate;	∞ , free stream flow.
Re_0 , Reynolds number, $\rho u d/\mu_0$;	
s , molecular speed ratio, $(\gamma/2)^{1/2} M$;	
T , temperature;	
T_A , measured adiabatic temperature;	
T_{aw} , true adiabatic probe temperature;	
u , velocity;	
x , distance along probe cylinder or distance along wake axis;	
α , energy accommodation coefficient;	
γ , specific heat ratio, c_p/c_v ;	
Δ , difference quantity;	

1. INTRODUCTION

IN MEASUREMENTS of total temperature in high speed (supersonic and hypersonic) flows many different probes have been proposed and used, shielded and unshielded thermocouple probes, [1-9] hot wires [10-12] and hot film probes [13].

However, in investigations of high speed boundary layers and wake flows, for example, where large total temperature gradients may occur, especially when the body is cooled, and where Reynolds number and Mach number vary from large values to zero in very small distances, problems arise with all these probes in measuring the total temperature accurately.

The hot wire probes break easily and at low Reynolds numbers heat conduction losses to the supports become large since the supports

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are at a temperature quite different from the recovery temperature of the hot wire. The necessary corrections may be as large as 10 per cent or more [11]. These heat losses also occur in hot film and thermocouple probes.

For thermocouple probes it has been tried to reduce this heat loss problem by heating the base and supports of the probes (Wood [3], McCarthy [5]). Designs like these make the probes quite complicated. The shielded thermocouple probes have the added problem of size. The smallest probes made so far are 0.5 mm at the tip [8] which is still too large for many measurements. Also, calibrations of these probes have only been made for continuum flow.

Because of these problems, a total temperature probe is suggested that combines the advantages of hot wire and thermocouple probe, reducing the heat conduction endlosses to a minimum but being simple in operation, and the data may be reduced easily. The principle of the probe is explained in the next section, followed by a description of a design. The data reduction procedure and endloss corrections are discussed

in Sections 4 and 5 and an example of the application of the probe is given in Section 6.

2. PRINCIPLE OF THE PROBE

The probe consists of a small thermally conducting circular cylinder which is placed normal to the oncoming stream. It is supported at its ends by thermally insulating material to minimize the heat exchange with the supports. A thermocouple junction is mounted on the cylinder halfway between the supports to measure the recovery temperature of the cylinder at that point ($x = 0$; the supports are at $x = \pm l/2$).

This type of cylinder probe was chosen for several reasons.

First, there is a large amount of data on the recovery factor as function of Knudsen number for fine metal cylinders [14-18]. For the present purposes—finding the total temperature of the flow—it is more convenient to use η as function of Reynolds number based on stagnation temperature and Mach number, a form of correlation already suggested by Morkovin [19]. This

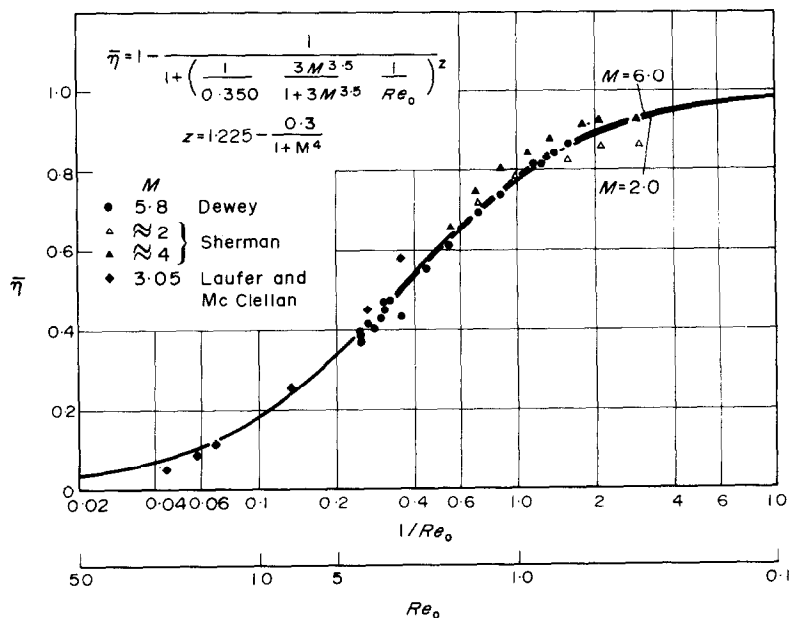


FIG. 1a. Recovery factor as function of Reynolds number and Mach number at $M \geq 2$.

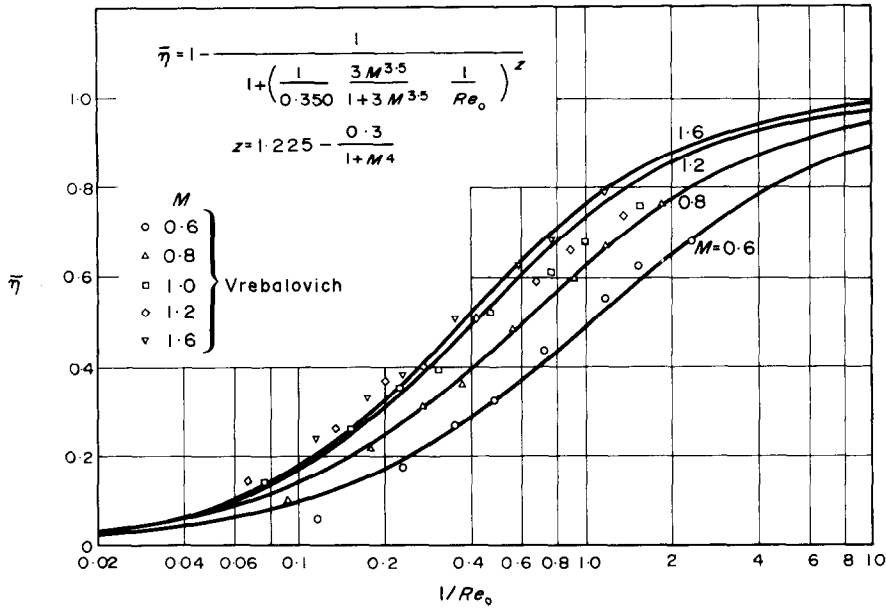


FIG. 1b. Recovery factor as function of Reynolds number and Mach number at $M < 2$.

correlation is shown in Figs. 1a and 1b, together with experimental data.

Second, the probe may be designed such that even in fairly low Reynolds number regions the temperature of the cylinder is close to the true recovery temperature and is not dominated by the temperature of the supports as in the case of a hot wire, hot film or shielded thermocouple probe.

Using the fact that at each cross-section x the temperature is constant across the cylinder, and assuming that the support temperatures are both T_s and the Nusselt number Nu_0 is constant in the region from support to support, i.e. from $x = \pm l/2$ and neglecting radiation, then the ratio between the true recovery temperature T_{aw} (without end losses) to the measured temperature T_A at the center of the probe ($x = 0$) is [10]

$$\frac{T_{aw}}{T_A} = \frac{\cosh \sqrt{(\alpha_0) - T_s/T_A}}{\cosh \sqrt{(\alpha_0) - 1}} \quad (1)$$

where

$$\alpha_0 = \frac{Nu_0}{a} = \frac{hd}{k_0} \frac{k_0(l/d)^2}{k_w} = \frac{hd}{k_w} (l/d)^2.$$

The cylinder probe supports are insulators and the temperature at each end of the cylinder should be close to the recovery temperature at the stagnation point of a cylinder ($\eta \approx 1$, see Dewey [18]). Therefore T_s/T_A is close to unity and very nearly the true recovery temperature of a conducting cylinder will be measured.

Furthermore, the cylinder diameter may be made an order of magnitude larger than a hot wire without sacrificing sensitivity (this feature is similar to the hot film probe [13]), thus making the Nusselt number larger [increasing α_0 , the heat conduction loss parameter, see equation (1)] which reduces the relative magnitude of the heat conduction to the supports. Heat losses caused by the thermocouple wires should also be negligible since they are mounted along the cylinder and are small compared to the cylinder.

Even if the temperature of the cylinder is non-uniform along the cylinder because of different support temperatures, the thermocouple measures the recovery temperature at the center between the supports, where the cylinder temperature is closest to the recovery tempera-

ture of an infinitely long wire. In case of a hot wire or hot film probe the mean temperature of the wire is measured. Thus, the endloss corrections for this type of probe are minimized by making the parameter α_0 as large as possible, $(T_s - T_A)/T_A$ as small as possible and by measuring the temperature T_A at the center between the supports and not the mean cylinder temperature.

Third, the probe may still be made small compared to a shielded thermocouple probe, but is as durable and does not have the heat loss problems of those probes in flows of low Reynolds number and/or large temperature gradients.

Fourth, since the probe consists of a thermally conducting cylinder, the temperature is constant at each cross section and the probe is directionally insensitive in two dimensions.

3. DESCRIPTION OF A CYLINDER PROBE DESIGN

Several probes were manufactured to measure the total temperature in the free stream at Mach 6 and in the base region behind a circular

needle (cylinder), 0.46 mm dia., is supported by micarta supports which are 25.4 mm apart. At the center between the supports two thermocouple wires (chromel-alumel) are welded to the needle (0.025 mm dia.). Another thermocouple is welded to the needle at one of the supports to obtain a measure of the temperature gradient along the cylinder for endloss-corrections if necessary. Of course a much smaller probe may be manufactured by butt-welding the thermocouple wires and using this welded thermocouple wire as cylinder, positioning the butt-welded junction halfway between the supports.

A somewhat more sophisticated probe, using the same principle, would be to make only a central portion of that cylinder out of a conductor (metal, to use the available data for the recovery factor) and extend it on both sides with thermally insulating cylinders. In that case also the supports would approach the same average temperature as the metal cylinder portion and thus further decrease the endlosses by decreasing $T_s - T_A$.

4. DATA REDUCTION PROCEDURE

The cylinder probe measures two temperatures: (1) the recovery temperature of the cylinder at the center between the supports, T_A ; (2) the support temperature, T_s . It is assumed that T_A is the true recovery temperature, T_{aw} , of the cylinder, an assumption which has to be checked (see next section). In order to find the total temperature of the flow, the recovery factor η has to be calculated.

The recovery factor η may be represented as function of Mach number and stagnation Reynolds number

$$Re_o = \frac{\rho u d}{\mu_o}$$

The available data on recovery factor of circular cylinders are represented by the empirical formulae (Figs. 1a and b):

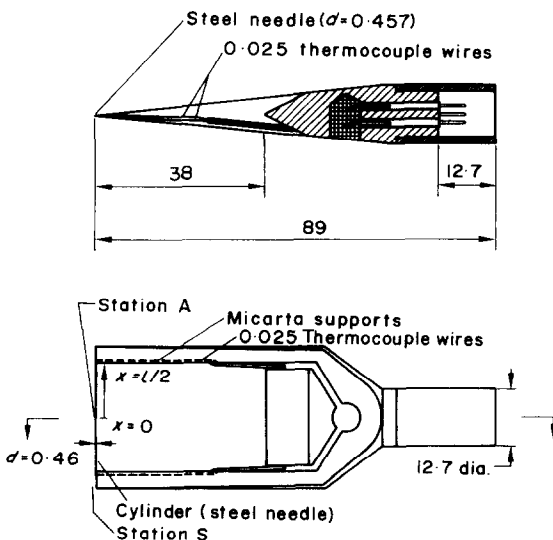


FIG. 2. Total temperature probe designed for wake flow measurements. All dimensions in mm.

cylinder, both adiabatic and cooled. The design of one of these probes is shown in Fig. 2. A

$$\bar{\eta} = \frac{\eta - \eta_c}{\eta_f - \eta_c} = 1 - \frac{1}{1 + \left[\frac{1}{0.350 Re_0 (1 + 3M^{3.5})} \right]^z} \quad (2)$$

where

$$z = 1.225 - \frac{0.3}{1 + M^4}$$

and

$$\eta_c = 1 - 0.050 \frac{M^{3.5}}{1.175 + M^{3.5}} \quad (3)$$

$$\eta_f - \eta_c = 0.2167 \frac{M^{2.8}}{0.8521 + M^{2.8}} \quad (4)$$

The correlations for η_c and $\eta_f - \eta_c$ were given by Dewey [18]. As $M \geq 2$ the relation

$$\eta = 1 - 0.217 \left/ \left[1 + \left(\frac{1}{0.350 Re_0} \right)^{1.225} \right] \right.$$

represents the data well. In order to find η , the quantities Re_0 and M have to be known, but η is only a rather weak function of Re_0 and also of M for $M \leq 2$, and is independent of M for $M > 2$.

To find Re_0 and M for practical data reduction purposes, distinguish three cases: Besides the temperature T_{aw} ,

(1) Pitot pressure and static pressure are known;

(2) Pitot pressure is known and Mach number $M \geq 2$;

(3) Mach number $M \leq 0.4$.

In the first case T_{aw} , p_p , p are measured. The ratio p/p_p yields M . For the purpose of finding Re_0 , let $T_{aw} \approx T_0$. Then

$$Re_0 = \rho u d / \mu_0 = (p M d / \mu_0) \sqrt{(T_0/T)} \sqrt{(\gamma/RT_0)}$$

The recovery factor η may be readily found using Figs. 1a and b or equations (2)–(4).

In the second case, T_{aw} and Pitot pressure are measured and $M \geq 2$; therefore $\eta = \eta(Re_0)$ and

only Re_0 has to be calculated. Again η is a rather weak function of Re_0 . In the region of strongest dependence of η on Re_0 , the transition regime from continuum to free molecule flow, when $Re_0 = 3$, say,

$$\frac{\Delta Re_0}{Re_0} = 12 \text{ per cent,}$$

then

$$\frac{\Delta \eta}{\eta} = 1 \text{ per cent.}$$

The dependence of η on Re_0 becomes weaker towards both the continuum and free molecule regime. When $Re_0 = 30$, $\eta = 0.96$ and $Re_0 \rightarrow \infty$, $\eta \rightarrow 0.95$. In order to find η , note that

$$\rho u^2 \approx 1.087 p_p / \left(1 + \frac{4}{15 M^2} \right); \text{ where } \gamma = 1.4$$

for $M \geq 2$ within 0.5 per cent. For $M \geq 3$ $\rho u^2 \approx 1.087 p_p$ within less than 4 per cent. Furthermore,

$$\frac{u}{u_\infty} \sqrt{(T_{0,\infty}/T_0)} = \frac{M}{M_\infty} \sqrt{\left(\frac{1 + [(\gamma - 1)/2] M_\infty^2}{1 + [(\gamma - 1)/2] M^2} \right)}$$

Any reasonable estimate of the Mach number will yield

$$\frac{u}{u_\infty} \sqrt{(T_{0,\infty}/T_0)}$$

quite accurately.

With the knowledge of ρu^2 ; $T_0/T_{0,\infty}$ and

$$\frac{u}{u_\infty} \sqrt{(T_{0,\infty}/T_0)},$$

the Reynolds number Re_0 may be calculated.

At low Mach numbers $\eta \rightarrow 1.0$ for all Reynolds numbers (case 3).

For $M \leq 0.34$, $\eta_f - \eta_c \leq 0.01$, thus $\eta = 1.0$ for any Reynolds number. As M increases to 0.4, $\eta_f - \eta_c \approx 0.027$ and using $\eta = 1.01$ will yield T_0 within 1 per cent for all Reynolds numbers. For larger Mach numbers, both M and Re_0 should be found approximately.

5. ENDLOSS CORRECTIONS AND RADIATION LOSSES

The idea is to construct the probe in such a way that any correction to the measured temperature, T_A , is unnecessary. But a check should always be made. If a circular cylinder, placed normal to the gas flow, is unheated but has support temperatures different from the "true" recovery temperature, T_{aw} , of the cylinder, then the temperature ratio T_{aw}/T_A is given by equation (1) (Section 2).

To calculate the true recovery temperature T_{aw} , the measured temperatures T_A and T_s , the probe parameters k_w and l/d , the thermal conductivity of air, k_0 , at stagnation temperature, T_0 , and the Nusselt number Nu_0 , have to be known. The thermal conductivity, k_0 , is accurately enough determined using $T_A \approx T_0$.

The Nusselt number Nu_0 is a function of Re_0 and M (see Fig. 3) and may be calculated with $T_A \approx T_0$. The Nusselt number over the free molecule Nusselt number is [20]

$$\frac{Nu_0}{Nu_f} = \frac{1}{1 + \frac{Nu_f}{Nu_{f,H}} \left(\frac{Nu_{f,H}}{Nu_{0,H}} - 1 \right)}$$

The Nusselt number for high speed flow ($M \geq 2$) is only a function of Reynolds number:

$$Re_{0,H} = 5.26 Nu_{0,H} + 5.74 Nu_{0,H}^2$$

or

$$Nu_{0,H} = \sqrt{(0.17422 Re_0 + 0.20994)} - 0.4582.$$

The Nusselt numbers for free molecule flow are given in [14]. Using

$$\alpha = 1, Pr_0 = 0.7 \text{ and } \gamma = 1.4$$

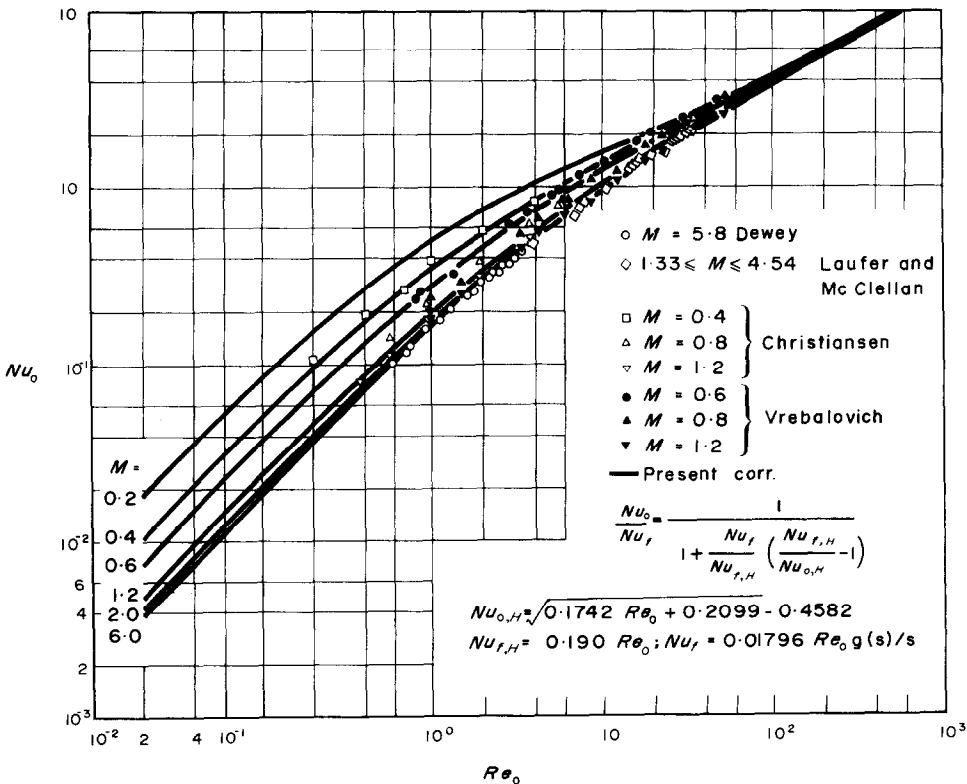


FIG. 3. Nusselt number-Reynolds number correlation for circular cylinder.

$$Nu_{f,H} = 0.190 Re_0$$

$$Nu_f = 0.01796 Re_0 \frac{g(s)}{s}$$

$$g(s) = 3(z_1 + z_2)$$

$$z_1 = \pi I_0 \left(\frac{s^2}{2} \right) \exp \left(-\frac{s^2}{2} \right)$$

$$z_2 = \pi s^2 \left[I_0 \left(\frac{s^2}{2} \right) + I_1 \left(\frac{s^2}{2} \right) \right] \exp \left(-\frac{s^2}{2} \right)$$

$$s = \sqrt{(\gamma/2) M}$$

The symbols I_0 and I_1 are modified Bessel functions of the first kind of zeroth and first order.

For very high temperatures, another source of error is the heat loss by radiation. An estimate of the temperature for which radiation becomes important is made. For a "point source" of radiation in a large room (like a small probe in a wind tunnel) the radiated heat per unit length of wire is

$$q_{rad} = \epsilon_w \pi d \sigma (T_w^4 - T_{iw}^4)$$

where

ϵ_w = emission ratio of wire

d = wire diameter

T_w = wire temperature

T_{iw} = tunnel wall temperature.

Considering the heat balance for the cylinder without endlosses, the heat radiated to the tunnel walls has to be equal to the heat transferred by convection to the probe

$$q_{conv.} = Nu_0 k_0 \pi (T_w - T_{aw}) = -q_{rad.}$$

Therefore the temperature change caused by radiation is

$$\frac{T_{aw} - T_w}{T_w} = \frac{\epsilon_w \sigma d (T_w^4 - T_{iw}^4)}{Nu_0 k_0 T_w}$$

This relation will give a good estimate at what temperature level radiation losses have to be

taken into account. For example, at a Nusselt number of 2, using a cylinder probe of steel ($\epsilon_w = 0.7$), at a total flow temperature of 800°K and a tunnel wall temperature of 700°K, the error would be about 3 per cent.

6. EXAMPLE OF TOTAL TEMPERATURE MEASUREMENTS

Temperature measurements with the cylinder total temperature probe shown in Fig. 2 were made by Hulcher and Behrens [21] in the near wake of a circular cylinder at Mach 6. The

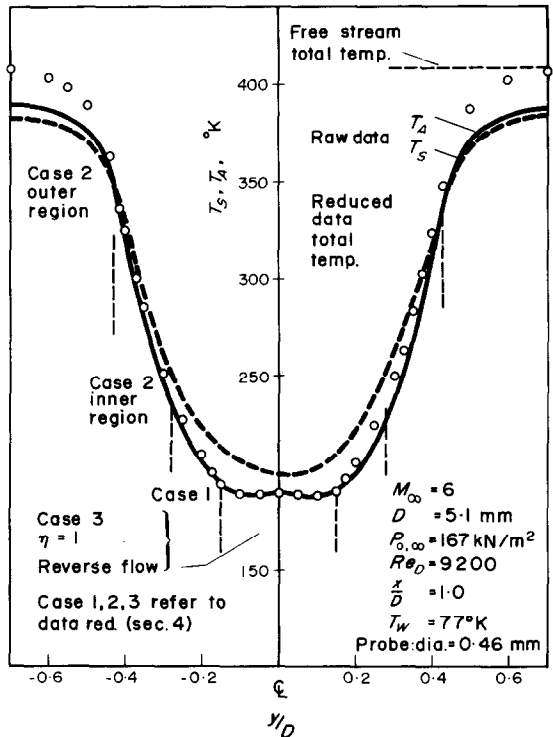


FIG. 4. Total temperature measurement in the near wake of a cylinder, cooled with liquid nitrogen.

cylinder model is cooled to 77°K with liquid nitrogen. In Fig. 4 temperature traces across the wake at $x/D = 1$, (measured from the center of the cylinder) are shown. T_A is the cylinder-probe temperature and T_s the probe support temperature (compare Fig. 2). The temperatures, T_A , are reduced to total temperatures, using the data

reduction procedures described in Section 4. For the data reduction, the Pitot pressure trace across the wake and the static pressure on the wake centerline were used. The region near the wake centerline is in the recirculation zone, the Mach number is $M \leq 0.3$ and $\eta = 1$ (case 3). In the region up to $y/D = 0.3$, the Mach number is calculated from p/p_p considering $p = \text{constant}$ across the thin shear layer and the data reduction proceeds as described under case 1. Near the edge of the shear layer $M = 2.5$ and the data reduction was done, using only p_p and T_A (case 2). The Mach number was estimated to be $M = 2.5$ and 3.5. Both Mach numbers resulted in the same η ($Re_0 > 30$). Heat conduction endloss corrections were made. Only towards the inner

edge of the shear layer and in the recirculation zone the corrections become necessary, since here $(T_s - T_A)/T_A$ becomes appreciable (see Fig. 5). However, this correction is simple and an iteration was not necessary. Another example of the use of the probe is given in [22], where total temperature measurements were performed in the near wake of a slender wedge, cooled with liquid nitrogen, at Mach number 4.

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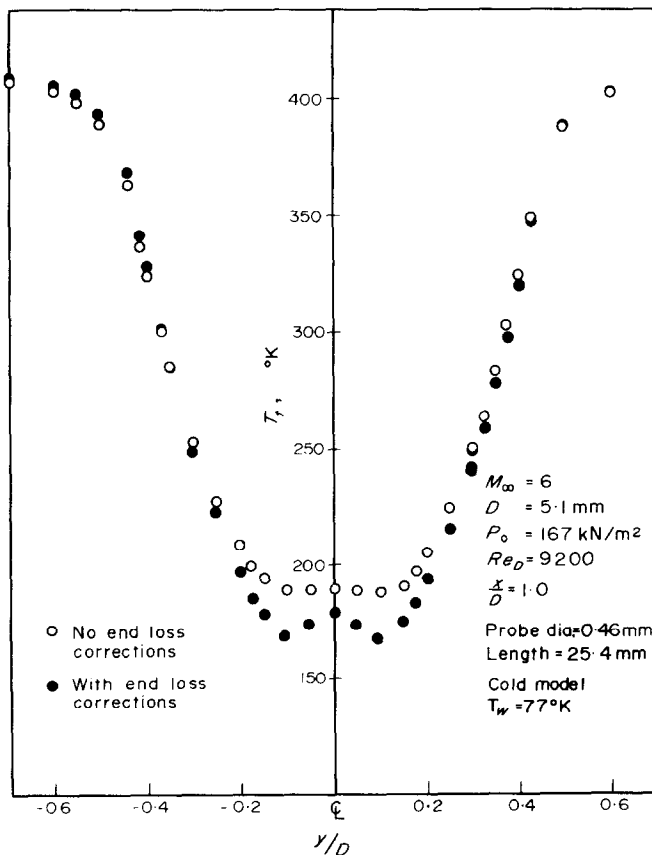


FIG. 5. Total temperature distribution in cold wake with and without endloss corrections.

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SONDE-THERMOCOUPLE POUR TEMPÉRATURE D'ARRÊT BASÉE SUR LA TEMPÉRATURE DE RÉCUPÉRATION D'UN CYLINDRE CIRCULAIRE

Résumé— Une sonde à température d'arrêt basée sur la température de récupération d'un cylindre circulaire est décrite. Elle combine les avantages du fil chaud et des sondes thermocouples à écran. Cette méthode est très simple, durable et peut être utilisée dans les écoulements lents ou rapides depuis le domaine continu jusqu'au régime moléculaire libre et convient pour les explorations de sillage et de couche limite avec de forts gradients de température d'arrêt. En exemple on discute des mesures de température d'arrêt dans l'hypersonique près du sillage d'un cylindre froid (77°K).

THERMOELEMENTSONDE ZUR MESSUNG DER RUHETEMPERATUR AUF DER BASIS DER EIGENTEMPERATUR EINES KREISZYLINDERS

Zusammenfassung— Es wird eine Ruhetemperatursonde auf der Basis der Eigentemperatur eines Kreiszyllinders beschrieben, die sowohl die Vorteile einer Hitzdrahtsonde als auch einer strahlungsgeschützten Thermoelementsonde vereint. Sie ist mechanisch sehr einfach und haltbar, kann bei niedriger und hoher Geschwindigkeit vom Bereich der Kontinuumsströmung bis zum Bereich der freien Molekülströmung verwendet werden und ist geeignet für Untersuchungen mit hohen Ruhetemperaturgradienten im Totwasser und in der Grenzschicht. Als Anwendungsbeispiel der Sonde werden Messungen der Ruhetemperatur in der hypersonischen Umgebung des Totwassers eines kalten Zylinders (77 K) erörtert.

КОМБИНИРОВАННЫЙ ТЕРМОПАРНЫЙ ДАТЧИК ТЕМПЕРАТУР,
ИСПОЛЬЗУЮЩИЙ ПРИНЦИП ТЕМПЕРАТУРЫ ВОССТАНОВЛЕНИЯ
КРУГОВОГО ЦИЛИНДРА

Аннотация—Описывается комбинированный термопарный датчик температуры, использующий принцип температуры восстановления кругового цилиндра, который соединяет в себе преимущества нагретой проволоки и экранированной термопары. Конструкция его очень проста и долговечна, он может быть использован в низко-и высокоскоростных течениях от континуума до свободномолекулярного режима и подходит для исследований следа и пограничного слоя при больших градиентах температуры. В качестве примера использования датчика рассматриваются измерения осредненной температуры в гиперзвуковом ближнем следе холодного цилиндра (77°K).